Monopoles, bions, and other oddballs in confinement or conformality

Erich Poppitz — U of Toronto

with Mithat Ünsal — SLAC/Stanford

things one would like to understand about any theory:

- does it confine?
- does it break its (super) symmetries?
- is it conformal?
- what are the spectrum, interactions...?

tough to address, in almost all theories

but relevant:

to satisfy curiosity, as well as for QCD and SUSY or non-SUSY extensions of the Standard Model conventional wisdom:

SUSY	- very "friendly" beautiful - exact results
pure YM	- formal but see www.claymath.org/millennium/
QCD-like	- hard, leave it to lattice folks chiral limit $$$
non-SUSY chiral gauge theories	- even lattice not practical ...nobody talks about them anymore

what I'll talk about applies to any of the above theories

have we solved these theories?

...

we use older and more recent results to study a regime where the nonperturbative dynamics of 4-d gauge theories - SUSY or not, chiral or vectorlike - is analytically tractable

compactifying 4d gauge theories on a small circle is a "deformation" where nonperturbative dynamics is under theoretical control

- as "friendly" as SUSY, e.g. Seiberg-Witten theory

in SUSY theories, "circle deformation" was pursued in late'90s:
then "forgotten", even within SUSY theory space

Seiberg, Witten (N=2 SYM)
Aharony, Hanany, Intriligator, Seiberg, Strassler;
Davies, Hollowood, Khoze, Mattis (N=1 SYM/SQCD)

a "revival" has occurred recently - both in SUSY and non-SUSY

Unsal; Unsal, Yaffe; Unsal, Shifman; Unsal, EP (2007-2009)

punchline:
we gain new, sometimes (perhaps) surprising, insight into the physics of confinement and abelian or discrete chiral symmetry breaking in vectorlike and chiral gauge theories with massless fermions
- all in a "locally 4d" setting

however... - "friendliness" on $R^3 \times S^1$ does not extend to R^4
... except very few special cases, not all SUSY

do not expect to compute detailed properties of QCD, or other theories

some qualitative information on the phase structure of the theories is likely to be relevant, however

we will attempt to "estimate" the critical number of massless fermion species where a gauge theory becomes conformal

perhaps surprisingly, we will see that results of very different uncontrolled calculations agree reasonably well with each other & with "experiment" (i.e. lattice, whenever available)

The plan

of this talk is to tell you, largely in pictures, what the above statements amount to.

time ↑ rigor ↓

Conformality or confinement (II): One-flavor CFTs and mixed-representation QCD
JHEP 0912:011,2009; 0910.1245, 33pp

Conformality or confinement: (IR)relevance of topological excitations
JHEP 0909:050,2009; 0906.5156, 42pp

Chiral gauge dynamics and dynamical supersymmetry breaking
JHEP 0907:060,2009; 0905.0634, 31pp

Index theorem for topological excitations on R*3 x S*1 and Chern-Simons theory
JHEP 0903:027,2009; 0812.2085, 29pp

(all by M. Unsal and E.P.)

First, the key players:

3d Polyakov model & "monopole-instanton"-induced confinement — Polyakov, 1977

"monopole-instantons" on $R^3 \times S^1$ — K. Lee, P. Yi, 1997; P. van Baal, 1998

the relevant index theorem — Nye, Singer, 2000; Unsal, EP, 2008

center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations — Shifman, Unsal, 2008; Unsal, Yaffe, 2008

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement — Unsal, 2007; Unsal, EP, 2009

First, the key players:

3d Polyakov model & "monopole-instanton"-induced confinement

Polyakov, 1977

continuum picture: 3d Georgi-Glashow [on the lattice - compact U(1)]

$$L \sim \frac{1}{g_3^2}\left(F^a_{\mu\nu} F^a_{\mu\nu} + D_\mu \phi^a D^\mu \phi^a\right) \qquad \mu,\nu = 1,2,3$$

$$[A_\mu] = [\phi] = 1 \qquad [g_3^2] = 1$$

due to some Higgs potential $\langle \phi \rangle = (0, 0, v)$

$SU(2) \xrightarrow{v} U(1)$ at low energies, $E \ll m_W \sim v$

free U(1) theory $A^3_\mu \equiv A_\mu$

$$\mathcal{L}_{eff} = \frac{1}{g_3^2} F_{\mu\nu}^2 + \ldots$$

"..." are perturbatively calculable
& not very interesting

$$B_\mu = \epsilon_{\mu\nu\lambda} F_{\nu\lambda}$$

"magnetic field"
topologically conserved current of **"emergent topological U(1) symmetry"** responsible for conservation of magnetic charge

$$B_\mu = g_3^2 \, \partial_\mu \sigma$$

3d photon dual to scalar (as one polarization only)

Abelian duality

$$\partial_\mu B_\mu = 0 \quad \longleftrightarrow \quad \partial_\mu^2 \sigma = 0$$

Bianchi identity $\longleftrightarrow$ equation of motion

$$\mathcal{L}_{eff} = \frac{1}{g_3^2} F_{\mu\nu}^2 + \ldots \qquad \mathcal{L}_{eff} = g_3^2 (\partial_\mu \sigma)^2 + \ldots$$

topological U(1) symmetry = shift of "dual photon"

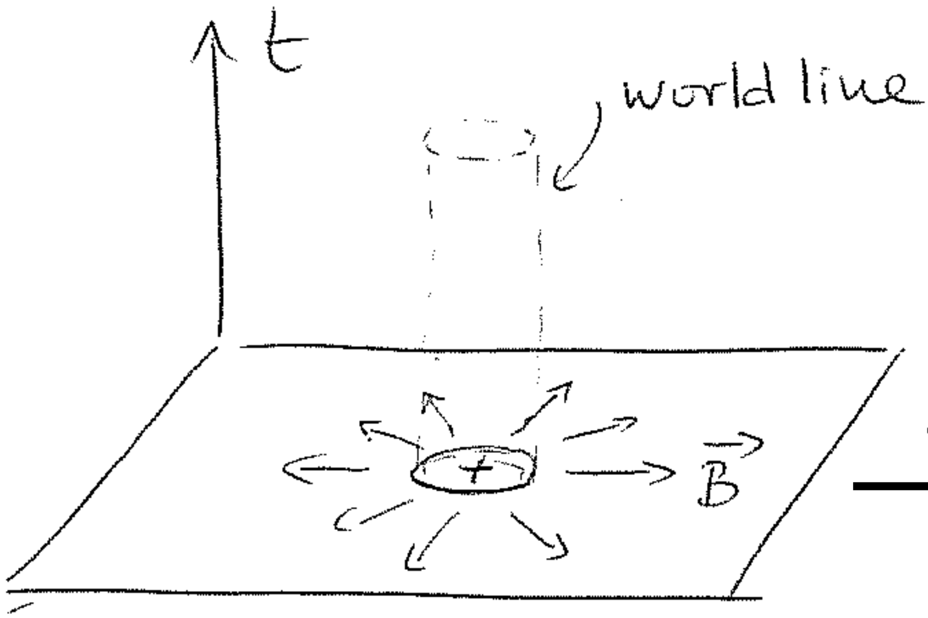
a rather ***"boring-boring" duality*** **-** if not for the existence of monopoles:

monopoles $\partial_\mu B_\mu$ = quantized magnetic charge - shift symmetry broken

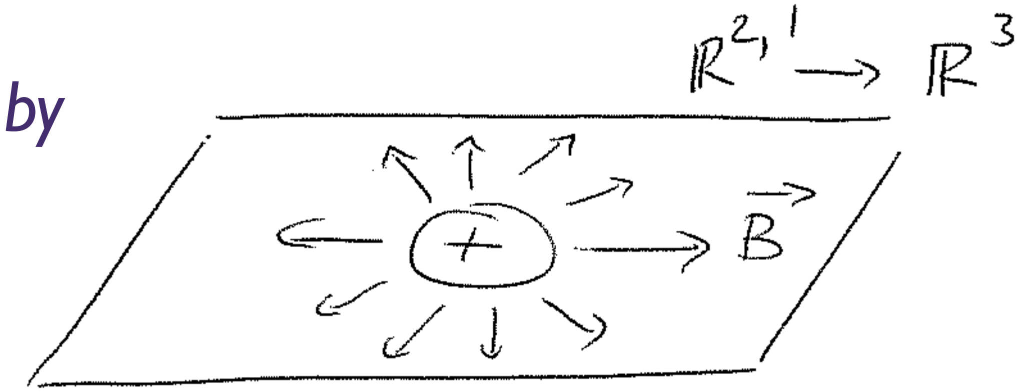
- dual photon gains mass & electric charges confined

how?
...in pictures:

“ ‘t Hooft-Polyakov monopole” - ***static finite energy solution of Georgi-Glashow model in 4d***



get Euclidean 3d by “forgetting time”



solution of Euclidean eqns. of motion of finite action: a “monopole-instanton”

$$E_M = \frac{4\pi v}{g_4^2} \longrightarrow S_0 = \frac{4\pi v}{g_3^2}$$

$$e^{-S_0} \rightarrow 0$$

$$g_3^2/v \rightarrow 0$$

M-M* pairs give exponentially suppressed (at weak coupling) “semiclassical” contributions to the vacuum functional

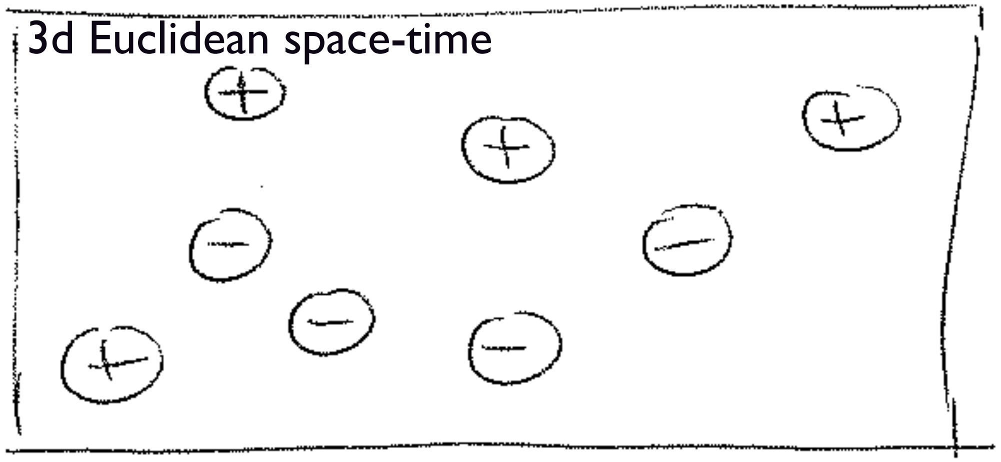
vacuum “is” a dilute monopole-antimonopole plasma

number of M-M* pairs per unit volume ~ $v^3 e^{-S_0}$

(analogous to B-L violation in electroweak theory - at T=0 exp. small, so no one cares!)

vacuum is a dilute M-M* plasma - but interacting, unlike instanton gas in 4d (in say, electroweak theory)

in pictures & in formulae



$Z =$
$Z(\text{perturbative}) \times$
$Z(\text{charged plasma with Coulomb interactions})$

really meaning grand partition function with fugacity $\exp(-S_0)$

physics is that of Debye screening; by analogy:

electric fields are screened in a charged plasma ("Debye mass for photon"), so in the monopole-antimonopole plasma, the dual photon obtains mass from screening of magnetic field:

$$\mathcal{L}_{eff} = g_3^2 (\partial\sigma)^2 + (\#)\, v^3 e^{-S_0} \left(e^{i\sigma} + e^{-i\sigma}\right) + \ldots$$

"(anti-)monopole operators"

aka **"disorder operators"** - not locally expressed in terms of original gauge fields

(Kadanoff-Ceva; 't Hooft - 1970s)

also by analogy with Debye mass:

dual photon mass^2 ~ M-M* plasma density

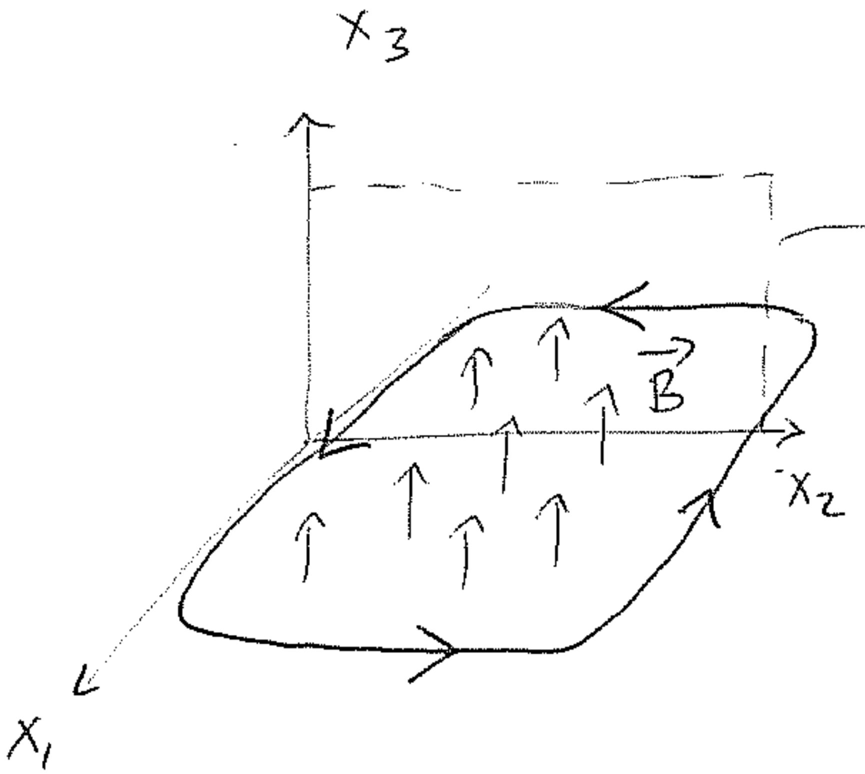
$$m_\sigma \sim v\, e^{-S_0/2} = v\, e^{-\frac{4\pi v}{2 g_3^2}}$$

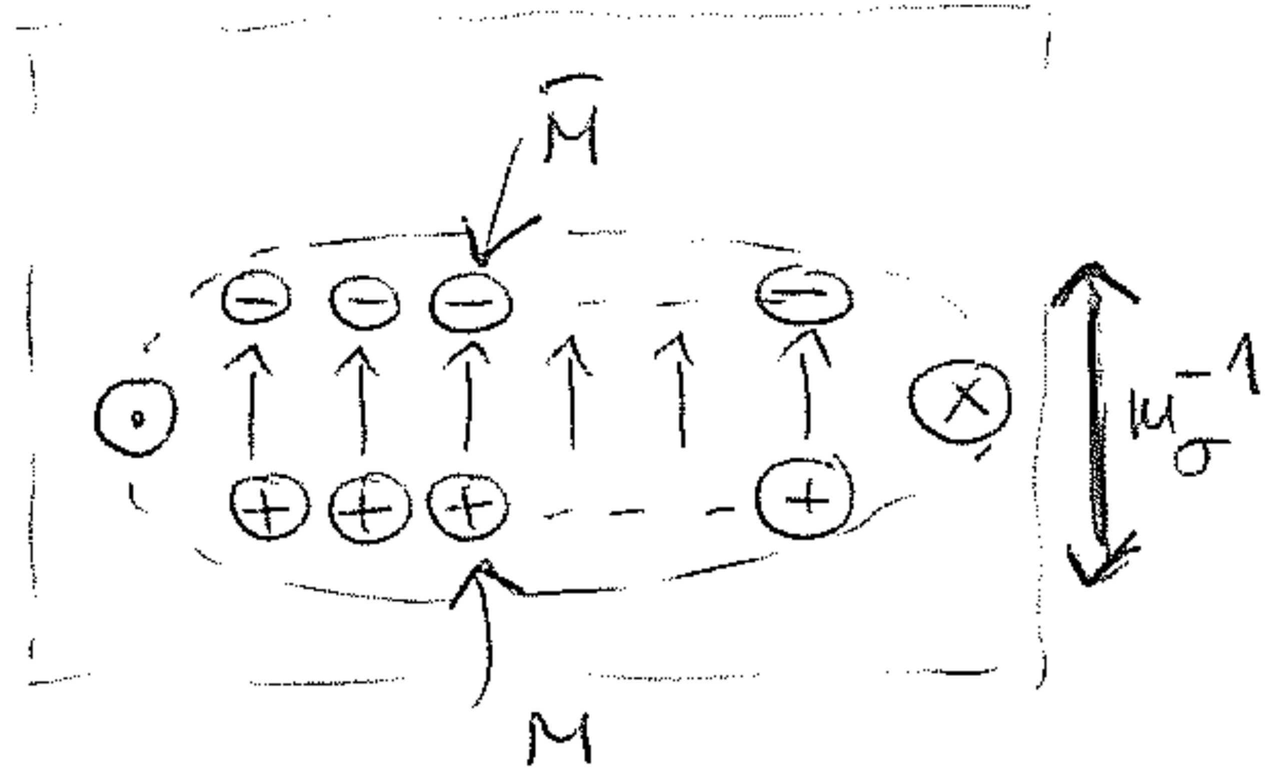
next:

dual photon mass

~ confining string tension...

in pictures:





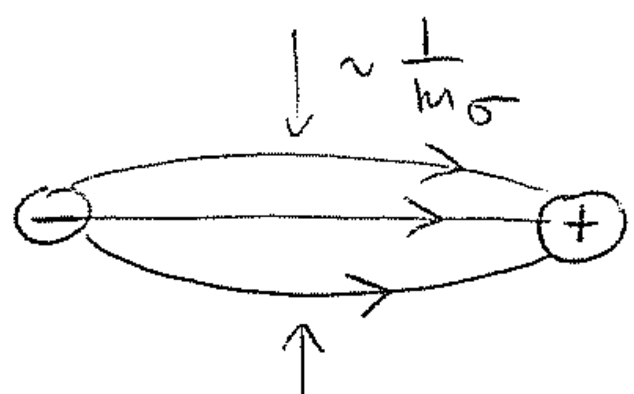
screening of magnetic field in plasma
= Wilson loop area law: $\langle e^{i \oint A dx} \rangle \sim e^{-(\text{Area})\, m_\sigma g_3^2}$

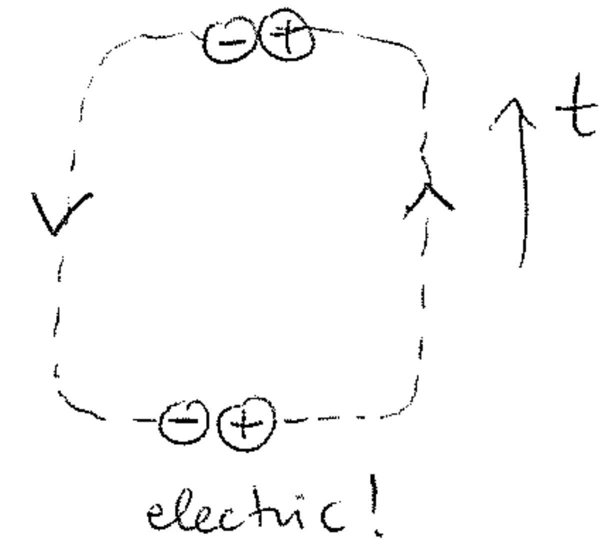
Minkowski space interpretation of Wilson loop: x_1 - time

$$0 \neq B^3 = \epsilon^{312} F^{12}$$

$$0 \neq \epsilon^{302} F^{02}$$

electric field





confining flux tube: **tension^{-1} ~ thickness ~ inverse dual photon mass**

First, the key players:

3d Polyakov model & "monopole-instanton"-induced confinement — Polyakov, 1977

"monopole-instantons" on $R^3 \times S^1$ — K. Lee, P. Yi, 1997; P. van Baal, 1998

the relevant index theorem — Nye, Singer, 2000; Unsal, EP, 2008

center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations — Shifman, Unsal, 2008; Unsal, Yaffe, 2008

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement — Unsal, 2007; Unsal, EP, 2009

First, the key players:

we want to go to 4d - by
"growing" a compact dimension: $S^1: \quad x^4 \sim x^4 + L$

"monopole-instantons" on $R^3 \times S^1$

K. Lee, P. Yi, 1997
P. van Baal, 1998

A_4 is now an adjoint 3d scalar Higgs field $\quad \partial_4 + A_4 \longrightarrow \frac{2\pi n}{L} + A_4$

but it is a bit unusual -
a compact Higgs field: $\langle A_4 \rangle \sim \langle A_4 \rangle + \frac{2\pi}{L}$ such shifts of A_4 vev absorbed into shift of KK number "n"

thus, natural scale of "Higgs vev" is $\langle A_4 \rangle \sim \frac{\pi}{L}$ leading to $SU(2) \xrightarrow{\frac{1}{L}} U(1)$

A_4 - adjoint 3d scalar Higgs field;
a gauge-covariant description:

$$W = P e^{i \oint_{S_1} A_4 \, dx^4}$$

"holonomy" around circle or "Polyakov loop"
- a unitary gauge-group element
- eigenvalues lie on unit circle
- trace of Polyakov loop is gauge invariant

if the expectation values are

$$\langle W \rangle = \begin{pmatrix} e^{i\pi/2} & \\ & e^{-i\pi/2} \end{pmatrix}$$

then $\mathrm{tr} \langle W \rangle = 0$

and we say that *"center symmetry is preserved"*

"center symmetry" = global symmetry of the theory on the circle, under which

$$\mathrm{tr}\, W \to e^{i\pi} \, \mathrm{tr}\, W \qquad \text{for SU(N): } e^{i\frac{2\pi}{N}}$$

"center symmetry" = symmetry associated with confinement in thermal compactifications, i.e. when L ~ inverse temperature:

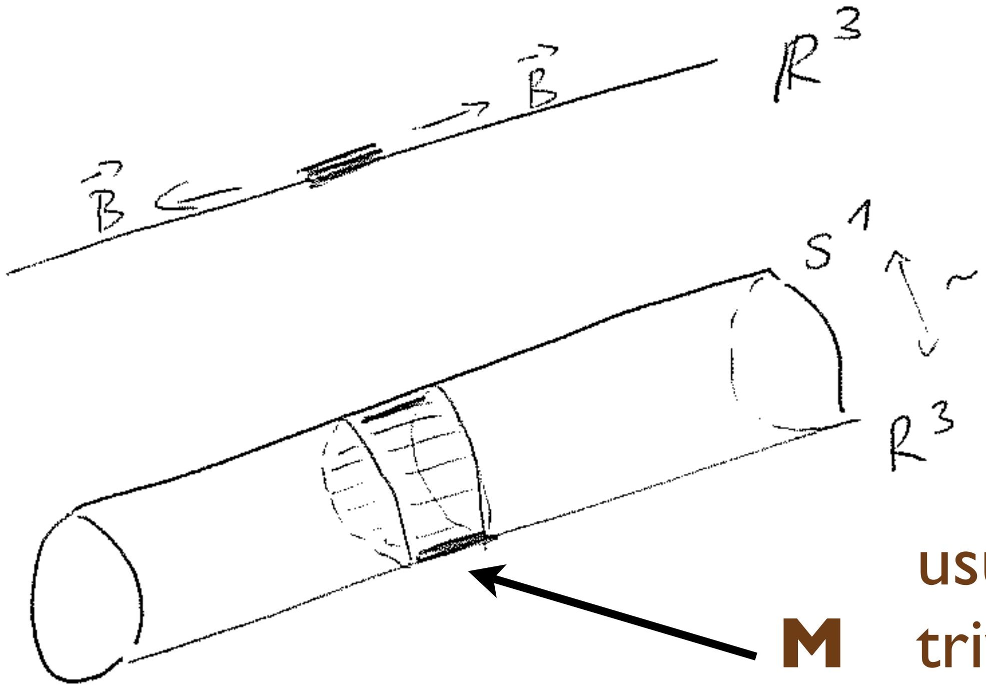
broken center = deconfinement

unbroken center = confinement $\quad \langle \mathrm{tr}\, W \rangle = e^{-\frac{F_q}{T}}$

we are interested in **unbroken center** cases: where <trW>=0 and SU(2) broken to U(1)

$$\langle W \rangle = \begin{pmatrix} e^{i\pi/2} & \\ & e^{-i\pi/2} \end{pmatrix}$$

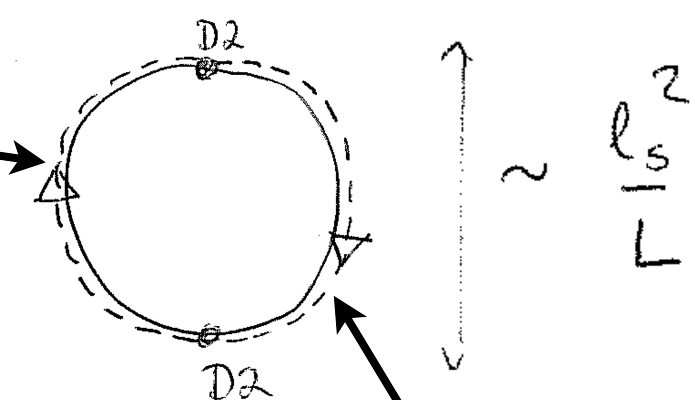
breaks SU(2) to U(1) so there are monopoles:



M usual monopole trivially embedded in 4d

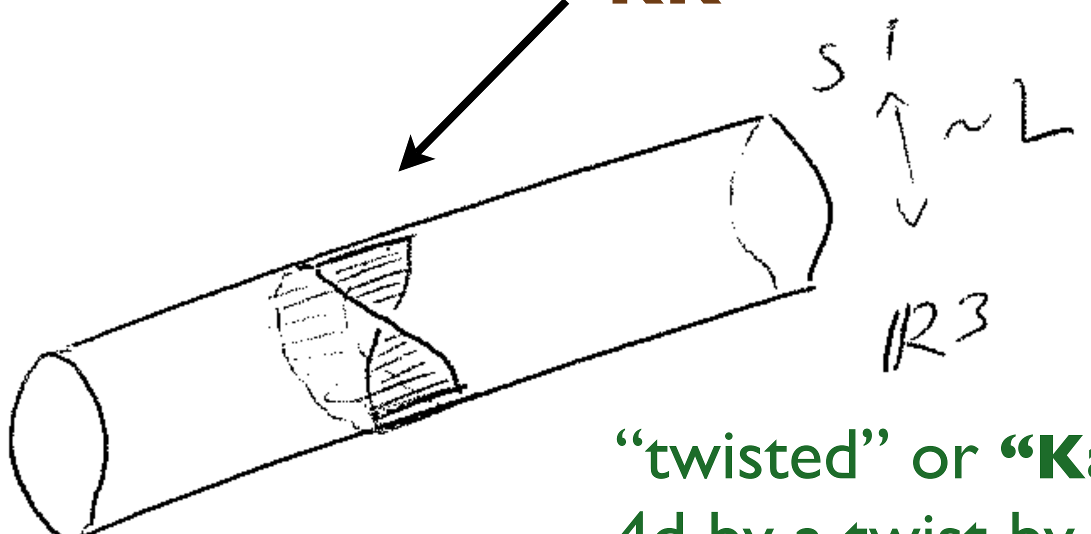
KK discovered by K. Lee, P. Yi, 1997, as *"Instantons and monopoles on partially compactified D-branes"*

M Euclidean D0-brane



KK Euclidean D0-brane

KK



"twisted" or **"Kaluza-Klein":** monopole embedded in 4d by a twist by a "gauge transformation" periodic up to center - in 3d limit not there! (infinite action)

X is the "Higgs field" of maximal abelian gauge

V are not well defined. Then V has a line of directional singularities, which one can interpret as the world line of a magnetic monopole (in euclidean space). In the generic case the world lines intersect the three dimensional region Ω in a discrete set of points. Because the eigenvalues of X are ordered, only adjacent pairs can become degenerate. If $\lambda_i = \lambda_{i+1}$, we shall label such a point $x^{(i)}$. Should X be an element of the group SU(N), one must keep in mind that (2.9) also admits $\lambda_1 = \lambda_N$ with $\phi_1 = \phi_N + 2\pi$. We shall label such points $x^{(0)}$ or $x^{(N)}$. Away from all $x^{(i)}$ the currents vanish because V is then differentiable often enough. It is therefore

Nuclear Physics B293 (1987) 461–478
North-Holland, Amsterdam

TOPOLOGY AND DYNAMICS OF THE CONFINEMENT MECHANISM

A.S. KRONFELD

Deutsches Elektronen-Synchrotron DESY, Hamburg, FRG

G. SCHIERHOLZ

Institut für Theoretische Physik, Universität Kiel, FRG
and Deutsches Elektronen-Synchrotron DESY, Hamburg, FRG

U.-J. WIESE

II. Institut für Theoretische Physik, Universität Hamburg, FRG

	magnetic charge	topological charge	semiclassical suppression
M	+1	1/2	e^{-S_0}
KK	−1	1/2	e^{-S_0}
BPST	0	1	e^{-2S_0}

both M & KK are self-dual objects, of opposite magnetic charges

+ their anti-"particles"

- thus, BPST instanton " = M+KK "

(aka "calorons" P. van Baal, 1998)

$$e^{-S_0} = e^{-\frac{4\pi v}{g_3^2}} = e^{-\frac{4\pi^2}{L g_3^2}} = e^{-\frac{4\pi^2}{g_4^2(L)}}$$

$$SU(N):\quad e^{-S_0} = e^{-\frac{8\pi^2}{g_4^2(L)\,N}}$$

(large-N survive!)

M & KK have, in SU(N), 1/N-th of the 't Hooft suppression factor aka: "fractional instantons", "instanton quarks", "zindons", "quinks", "instanton partons"... [collected by D. Tong]

Next, to understand the role M, KK, M* & KK* play in various theories of interest, need to know what happens to the operators they induce when there are fermions in the theory.

First, the key players:

3d Polyakov model & "monopole-instanton"-induced confinement — Polyakov, 1977

"monopole-instantons" on $R^3 \times S^1$ — K. Lee, P. Yi, 1997; P. van Baal, 1998

the relevant index theorem — Nye, Singer, 2000; Unsal, EP, 2008

center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations — Shifman, Unsal, 2008; Unsal, Yaffe, 2008

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement — Unsal, 2007; Unsal, EP, 2009

First, the key players:

the relevant index theorem — Nye, Singer, 2000; Unsal, EP, 2008

- for some theories the answer for the number of zero modes in M or KK background had been guessed (correctly)

  e.g. SUSY YM - Aharony, Hanany, Intriligator, Seiberg, Strassler, 1997

- while studying Intriligator-Seiberg-Shenker proposed model of SUSY breaking, Unsal and I needed a general index theorem [SU(2)+three-index symm. tensor Weyl]

- we found this:

Journal of Functional Analysis **177**, 203–218 (2000)
doi:10.1006/jfan.2000.3648, available online at http://www.idealibrary.com on IDEAL®

An L^2-Index Theorem for Dirac Operators on $S^1 \times \mathbb{R}^3$

Tom M. W. Nye and Michael A. Singer

where, in APPENDIX A. ADIABATIC LIMITS OF η-INVARIANTS

we found:

$$\text{ind}\,(D^+_{\mathbb{A}}) = \int_X \text{ch}(\mathbb{E}) + \frac{1}{\mu_0} \sum_\mu \epsilon_\mu c_1(E_\mu)[S^2_\infty]$$

$$= \int_X \text{ch}(\mathbb{E}) - \frac{1}{2}\overline{\eta}_{\text{lim}}$$

(last formula in paper)

two obvious questions:

1.) where does this come from?

2.) what number is it equal to in a given topological background (M,KK...)
& how does it depend on ratio of radius to holonomy?

for answers & more

see M. Unsal, EP

0812.2085

like on R^3 Callias $\xleftarrow{\text{physicist derivation}}$ E. Weinberg, 1970s, but on $R^3 \times S^1$, so must incorporate anomaly equation, some interesting effects

for this talk it is enough to consider 4d SU(2) theories with N_W adjoint Weyl fermions

"applications": $N_W = 1$ is pure N=1 SUSY YM

$N_W = 4$ some call it "minimal walking technicolor"; also happens to be N=4 SYM without the scalars

M KK M* KK* each have $2N_W$ zero modes

disorder operators:

M:

$$e^{-S_0} e^{i\sigma} (\lambda\lambda)^{N_W}$$

KK:

$$e^{-S_0} e^{-i\sigma} (\lambda\lambda)^{N_W}$$

M*:

$$e^{-S_0} e^{-i\sigma} (\bar\lambda\bar\lambda)^{N_W}$$

KK*:

$$e^{-S_0} e^{i\sigma} (\bar\lambda\bar\lambda)^{N_W}$$

where:

$$(\lambda\lambda)^{N_W} = \det_{\underset{SU(N_W)}{I,J}} \lambda^{\overset{SU(2)}{a}}_{\alpha I} \lambda^{a}_{\underset{SL(2,\mathbb{C})}{\beta} J} \epsilon^{\alpha\beta}$$

remarks:

- operator due to M+KK = 't Hooft vertex; independent of dual photon
- "our" index theorem interpolates between 3d Callias and 4d APS index thms.

First, the key players:

3d Polyakov model & "monopole-instanton"-induced confinement

Polyakov, 1977

"monopole-instantons" on $R^3 \times S^1$

K. Lee, P. Yi, 1997
P. van Baal, 1998

the relevant index theorem

Nye, Singer, 2000
Unsal, EP, 2008

center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations

Shifman, Unsal, 2008
Unsal, Yaffe, 2008

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement

Unsal, 2007
Unsal, EP, 2009

First, the key players:

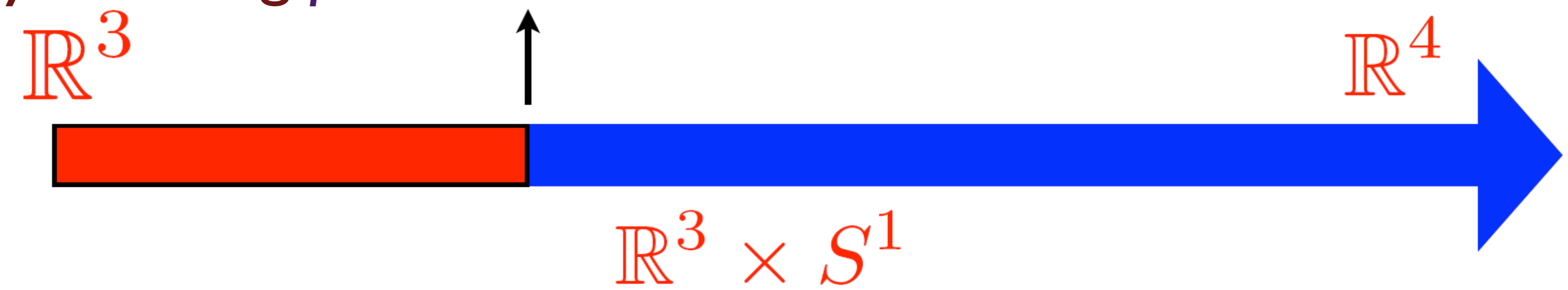
- Abelianization occurs only if there is a nontrivial holonomy (i.e., A_4 has vev)

- upon thermal circle compactifications, gauge theories with fermions do not Abelianize: center symmetry is broken at small circle size - transition to a deconfining phase - A_4=0, <trW>=0 - deconfinement - at high-T, 1-loop V_{eff} (Gross, Pisarski, Yaffe, early1980s)

center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations

Shifman, Unsal, 2008
Unsal, Yaffe, 2008

in other words, in thermal setup, upon decompactification, we have a center-symmetry breaking *phase transition* and no smooth connection to R^4



to ensure calculability at small L and smooth connection to large L in the sense of center symmmetry: *can one find ways to avoid phase transition?*

I. non-thermal compactifications - periodic fermions ("twisted partition function")

- with $N_W > 1$ adjoint fermions center symmetry preserved (Unsal, Yaffe 2007) as well as with other, "exotic" fermion reps (Unsal, EP 2009)
- in many supersymmetric theories, can simply choose center-symmetric vev

II. add double-trace deformations: force center symmetric vacuum at small L (Shifman, Unsal 2008)

In what follows, we assume center-symmetric vacuum - due to either **I**. or **II**. - will explicitly discuss only theory where center symmetry is naturally preserved at small L (**I**.)

First, the key players:

3d Polyakov model & “monopole-instanton”-induced confinement

Polyakov, 1977

“monopole-instantons” on $R^3 \times S^1$

K. Lee, P. Yi, 1997
P. van Baal, 1998

the relevant index theorem

Nye, Singer, 2000
Unsal, EP, 2008

center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations

Shifman, Unsal, 2008
Unsal, Yaffe, 2008

“bions”, “triplets”, “quintets”... - new non-self-dual topological excitations and confinement

Unsal, 2007
Unsal, EP, 2009

First, the key players:

ready to study the dynamics of theories with massless fermions on a small circle

in a vacuum with A_4 vev, Abelianization:

- in SU(2): (dual) photon massless + fermion components w/out mass from vev (neutral)
- monopoles + KK monopoles are the basic topological excitations

is there magnetic field screening in the vacuum?

the answer would appear to be "no":

M and KK have fermion zero modes

monopole operators do not generate potential for dual photon

so, **no screening & no confinement... ?**

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement

Unsal, 2007
Unsal, EP, 2009

but take a look at the symmetries first:

as an example, again consider 4d SU(2) theories with N_W adjoint Weyl fermions

classical global chiral symmetry is

$$SU(N_W) \times U(1)$$

but 't Hooft vertex $(\lambda\lambda)^{2N_W} e^{-\frac{8\pi^2}{g_4^2}}$ only preserves $\mathbb{Z}_{4N_W}: \lambda \to e^{i\frac{2\pi}{4N_W}} \lambda$

so, quantum-mechanically we have only $SU(N_W) \times Z_{4N_W}$ exact chiral symmetry

now **M, KK**(+*) operators all look like: $e^{-S_0} e^{i\sigma} (\lambda\lambda)^{N_W}$

hence $(\lambda\lambda)^{N_W} \xrightarrow{\mathbb{Z}_{4N_W}} e^{i\pi} (\lambda\lambda)^{N_W}$

invariance of **M**, **KK**(+*) operators under exact chiral symmetry means that

dual photon must transform under the exact chiral symmetry

i.e., topological shift symmetry is intertwined with chiral symmetry:

$$\mathbb{Z}_{4N_W}: \quad \sigma \to \sigma + \pi$$

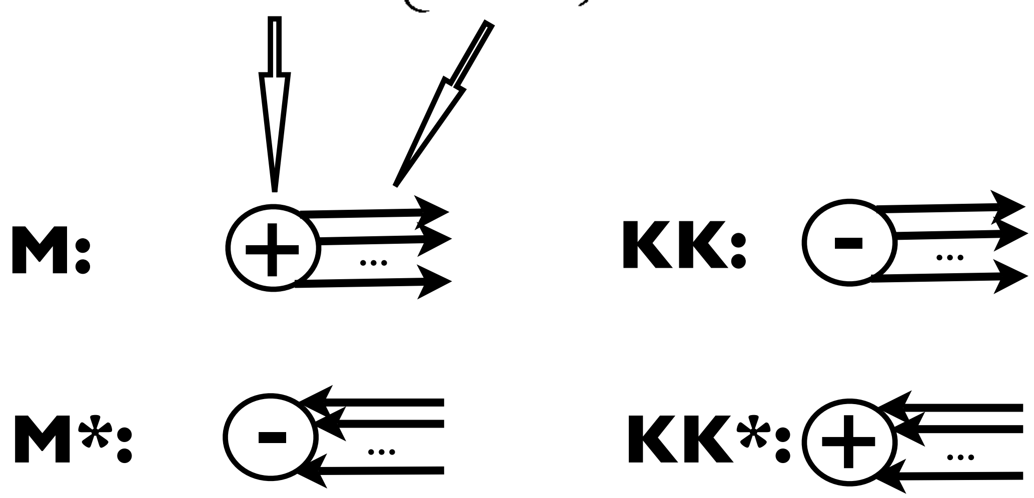
$$\sigma \to \sigma + \pi \qquad \cancel{\cos\sigma} \qquad \cos(2\sigma) \ \checkmark$$

so the exact chiral symmetry allows a potential - **but what is it due to?**

to generate $\cos(2\sigma)$ must have

i. magnetic charge 2
ii. no zero modes

$$e^{-S_0}\, e^{i\sigma}\, (\lambda\lambda)^{N_w}$$



M + KK* bound state? (Unsal, 2007)

- same magnetic charge ~ 1/r-repulsion
- fermion exchange ~ log(r)-attraction

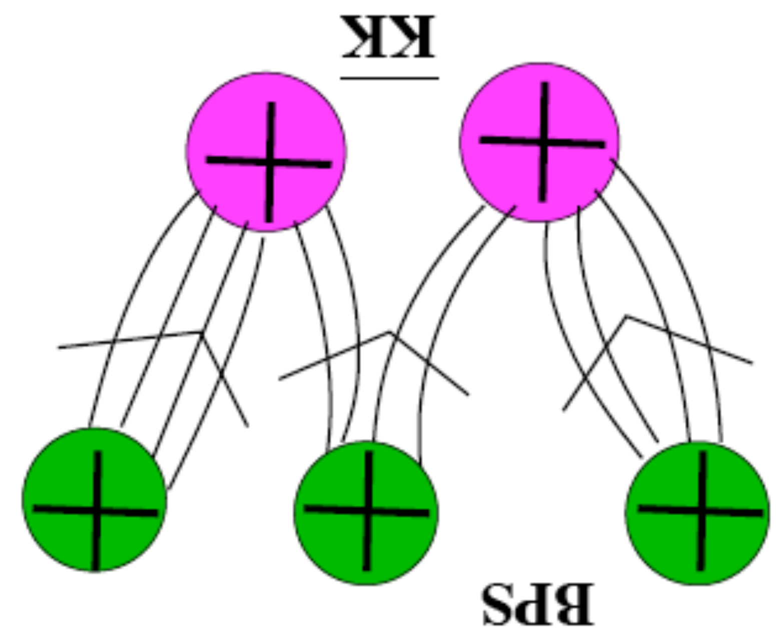
M + KK* = B - magnetic "bion"

$$e^{-2S_0}\left(e^{2i\sigma} + e^{-2i\sigma}\right)$$

dual photon mass induced by magnetic "bions"- the leading cause of confinement

to summarize, in QCD(adj),

cartoon of a “quintet”:



M + KK* = B - magnetic “bions” -

carry magnetic charge

no topological charge (non self-dual)

(locally 4d nature crucial: no KK in 4d)

generate “Debye” mass for dual photon

main tools

- intertwining of topological shift symmetry & chiral symmetry
- index theorem

topological objects generating magnetic screening depend on massless fermion content (not usually thought that fermions relevant)

using these tools, one can analyze any theory...

in the last couple of years, many theories have been studied...

vectorlike | chiral

Theory all SU(N)	Confinement mechanism on $\mathbb{R}^3 \times S^1$	Index for monopoles $[\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_N]$ Nye-M.Singer '00; PU '08	Index for instanton $I_{inst.} = \sum_{i=1}^{N} I_i$ Atiyah-Singer	$(\text{Mass Gap})^2$ units ~1/L^2
YM Y,U '08	monopoles	$[0, \ldots, 0]$	0	e^{-S_0}
QCD(F) S,U '08	monopoles	$[2, 0, \ldots, 0]$	2	e^{-S_0}
SYM /QCD(Adj) U '07	magnetic bions	$[2, 2, \ldots, 2]$	$2N$	e^{-2S_0}
QCD(BF) S,U '08	magnetic bions	$[2, 2, \ldots, 2]$	$2N$	e^{-2S_0}
QCD(AS) S,U '08	bions and monopoles	$[2, 2, \ldots, 2, 0, 0]$	$2N - 4$	e^{-2S_0}, e^{-S_0}
QCD(S) P,U '09	bions and triplets	$[2, 2, \ldots, 2, 4, 4]$	$2N + 4$	e^{-2S_0}, e^{-3S_0}
SU(2)YM $I = \frac{3}{2}$ P,U '09	magnetic quintets	$[4, 6]$ SUSY version: ISS(henker) model of SUSY [non-]breaking	10	e^{-5S_0}
chiral $[SU(N)]^K$ S,U '08	magnetic bions	$[2, 2, , \ldots, 2]$	$2N$	e^{-2S_0}
$AS + (N-4)\overline{\text{F}}$ S,U '08	bions and a monopole	$[1, 1, \ldots, 1, 0, 0]$ + $[0, 0, \ldots, 0, N-4, 0]$	$(N-2)\text{AS} + (N-4)\overline{\text{F}}$	e^{-2S_0}, e^{-S_0},
$S + (N+4)\overline{\text{F}}$ P,U '09	bions and triplets	$[1, 1, \ldots, 1, 2, 2]$ + $[0, 0, \ldots, 0, N+4, 0]$	$(N+2)\text{S} + (N+4)\overline{\text{F}}$	e^{-2S_0}, e^{-3S_0},

name codes:

U=Unsal
S=Shifman
Y=Yaffe
P=the speaker

Table 1. Topological excitations which determine the mass gap for gauge fluctuations and chiral symmetry realization in vectorlike and chiral gauge theories on $\mathbf{R}^3 \times \mathbf{S}^1$. Unless indicated otherwise,

+ SO(N),SP(N) - S. Golkar 0909.2838; for mixed-representation/higher-index reps. SU(N) - P,U 0910.1245

So, I have now introduced all the key players:

3d Polyakov model & "monopole-instanton"-induced confinement	Polyakov, 1977
"monopole-instantons" on $R^3 \times S^1$	K. Lee, P. Yi, 1997 P. van Baal, 1998
the relevant index theorem	Nye, Singer, 2000 Unsal, EP, 2008
center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations	Shifman, Unsal, 2008 Unsal, Yaffe, 2008
"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement	Unsal, 2007 Unsal, EP, 2009

The upshot is the **dual lagrangian of QCD(adj)** on a circle of size L:

$$\frac{g^2(L)}{2L}(\partial\sigma)^2 - \frac{b}{L^3}\, e^{-2S_0}\cos 2\sigma + i\bar{\lambda}^I\gamma_\mu\partial_\mu\lambda_I + \frac{c}{L^{3-2N_f}}\, e^{-S_0}\cos\sigma(\det_{I,J}\lambda^I\lambda^J + \text{c.c.})$$

B, B*

M, KK+*

leading-order perturbation theory; perturbative corrections ~ $g_4(L)^2$ omitted

$$m_\sigma \sim \frac{1}{L} e^{-S_0} = \frac{1}{L} e^{-\frac{8\pi^2}{N_c g_4^2(L)}}$$

$$(\Lambda L)^{\beta_0} = e^{-\frac{8\pi^2}{g_4^2(L)}}$$

$$\beta_0 = \frac{11}{3} N_c - \frac{2}{3} N_w N_c$$

$$m_\sigma = \frac{1}{L}(\Lambda L)^{\frac{\beta_0}{N_c}} = \Lambda(\Lambda L)^{\frac{\beta_0}{N_c}-1} = \Lambda(\Lambda L)^{\frac{8-2N_w}{3}}$$

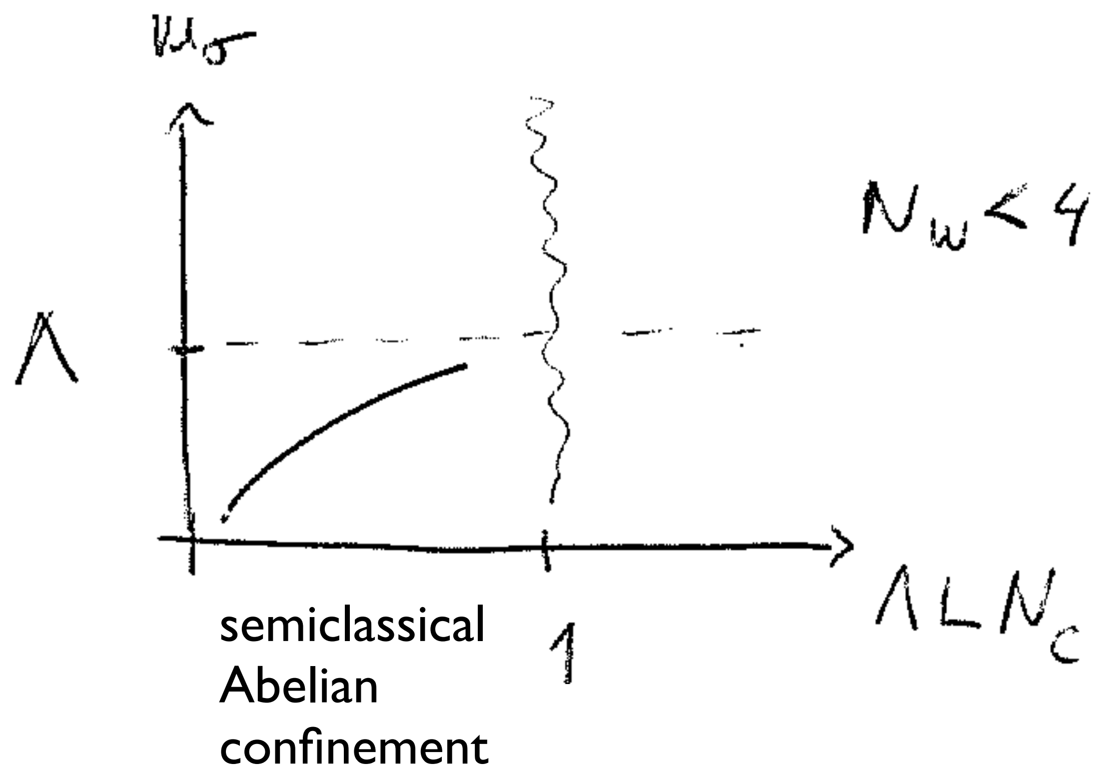
mass gap ~ string tension behaves in an interesting way

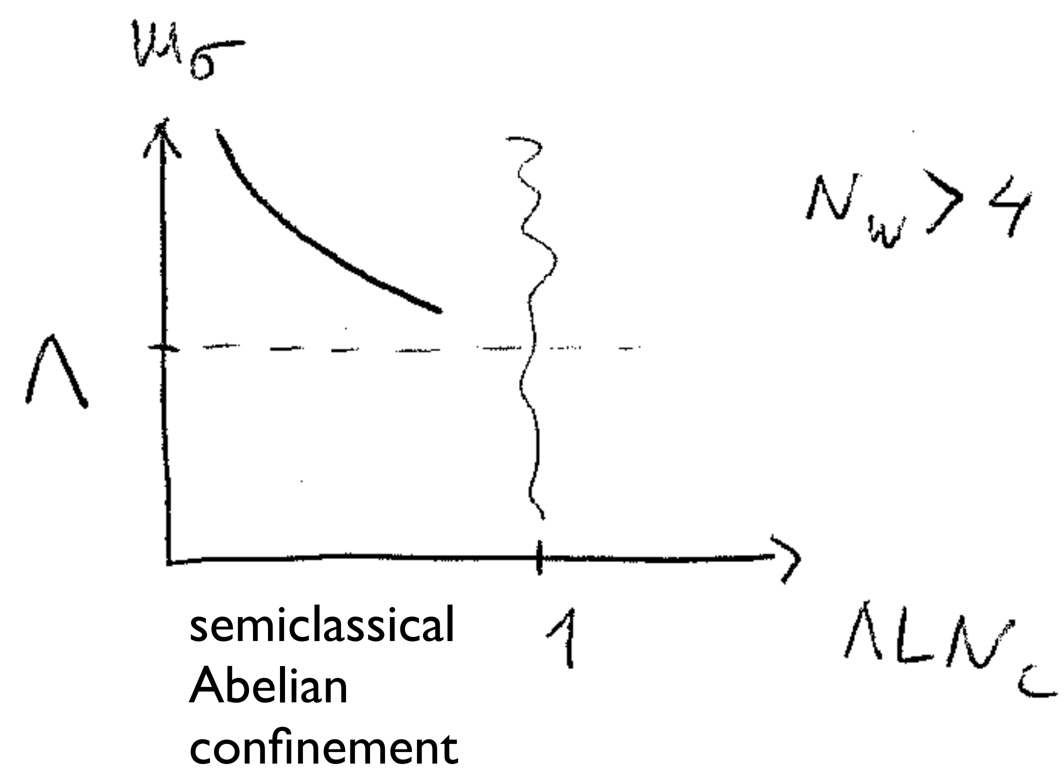
as L changes at fixed Λ ... $N_w^* = 4$?

region of validity of semiclassical analysis:

$\Lambda L \ll 1$ ($N_c \Lambda L \ll 1$, really) as mass of W ~ 1/(NL)

$$m_\sigma \sim \Lambda (\Lambda L)^{(8-2N_w)/3}$$





analysis shows that this switch of behavior as number of fermion species is increased occurs in all theories - vectorlike or chiral alike

in each case we obtain a value for the critical number of "flavors" or "generations"... N_f^*

like $N_w^* = 4$ for QCD(adj)

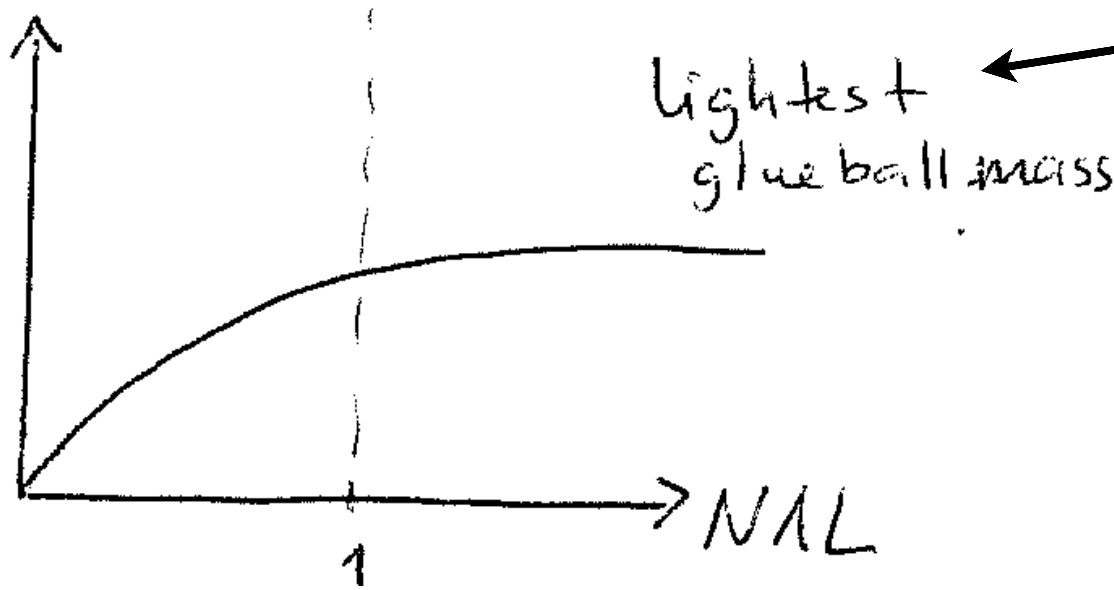
does it tell us anything about R^4?

(what follows is the promised not-so-rigorous part) →

I know I am in danger of being arrested...

... how **dare** you study non-protected quantities?

A reasonable expectation of what happens at very small or very large number of "flavors" is this:



topological excitations become non-dilute with increase of L, cause confinement, M, KK+* operators

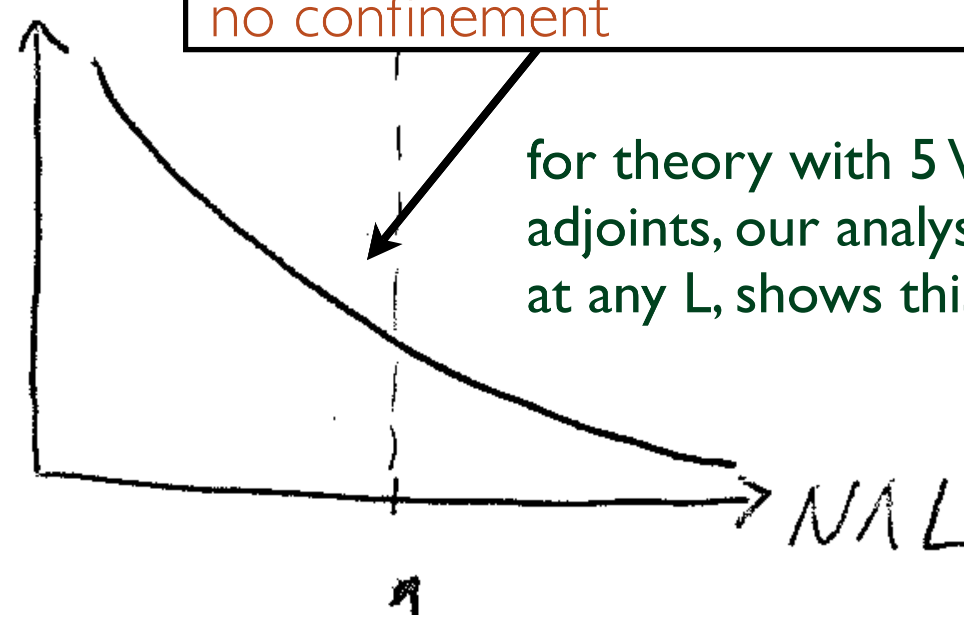
$$e^{-S_0} \cos\sigma (\det_{I,J} \lambda^I \lambda^J + \text{c.c.})$$

become strong, can cause chiral symmetry breaking (whenever the confining theories break their nonabelian chiral symmetries)

sufficiently small # fermion species
confining theories

sufficiently large # fermion species
fixed point at weak coupling
conformal in IR, no mass gap

topological excitations that cause confinement dilute with increase of L, no confinement



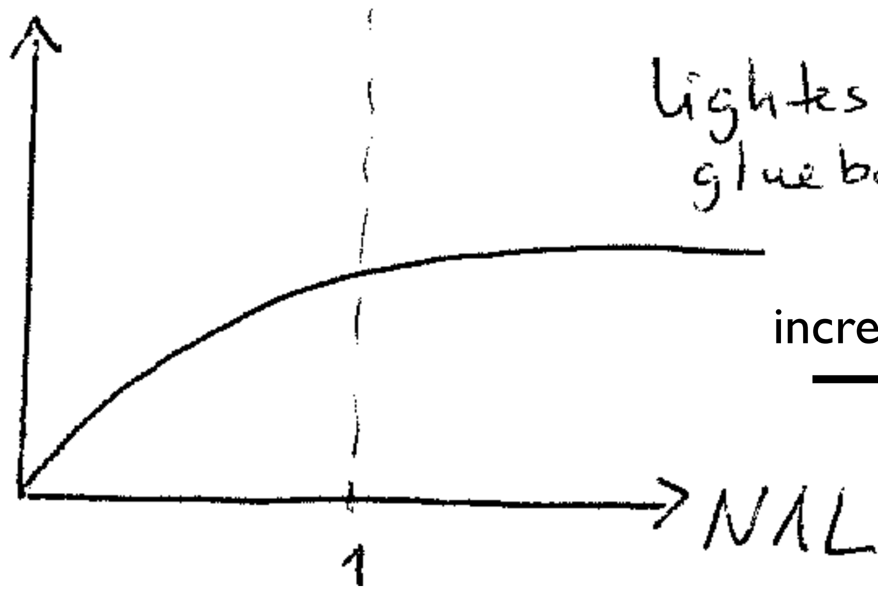
for theory with 5 Weyl adjoints, our analysis is valid at any L, shows this behavior

but where does the transition **really** occur?
is it at our value N_f*?

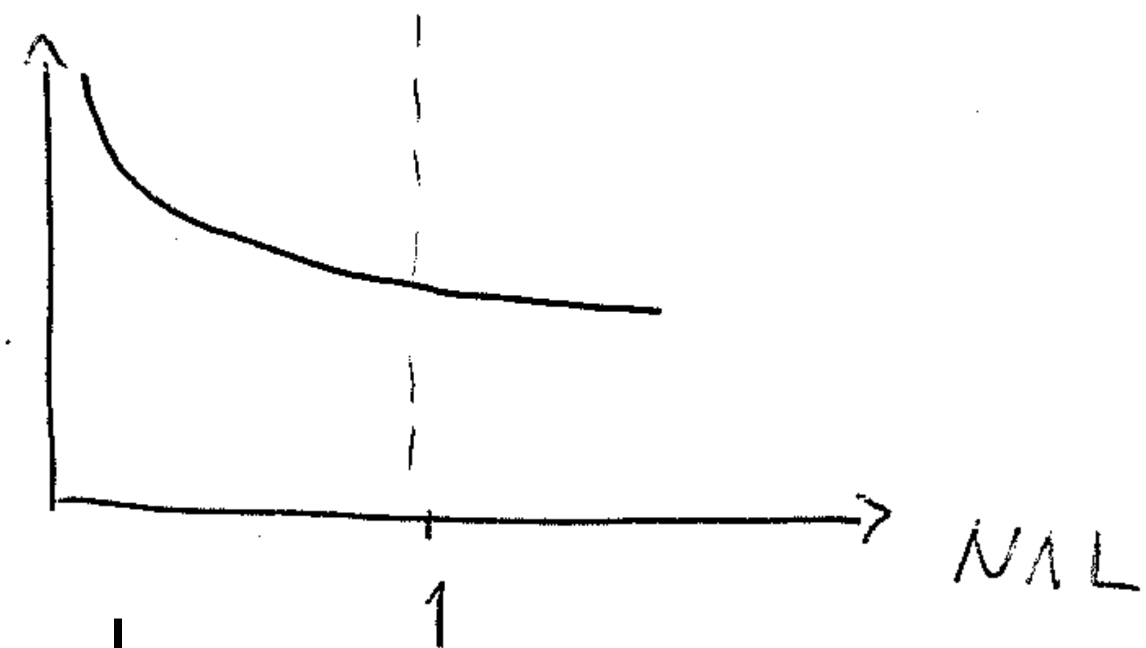
there appear to be three possibilities
(in any given class of theories, only one is realized)

A.) our N_f* is the true critical value N_{crit} [theory that may be in this class: QCD(adj), experiment (lattice)]

B.) if, as # species is increased above N_f^*

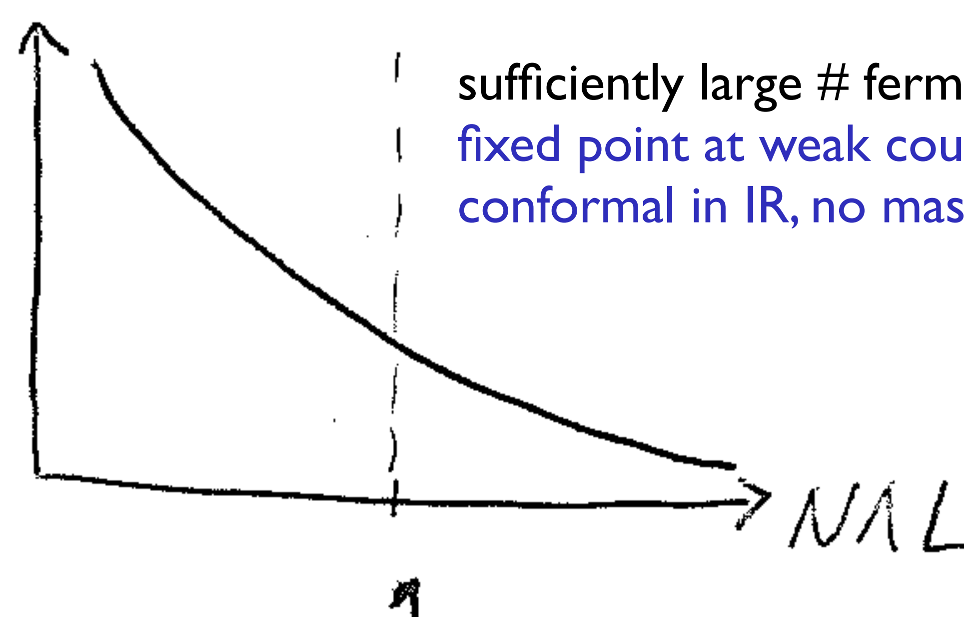


increase # fermions



sufficiently small # fermion species
confining theories

increase # fermions



sufficiently large # fermion species
fixed point at weak coupling
conformal in IR, no mass gap

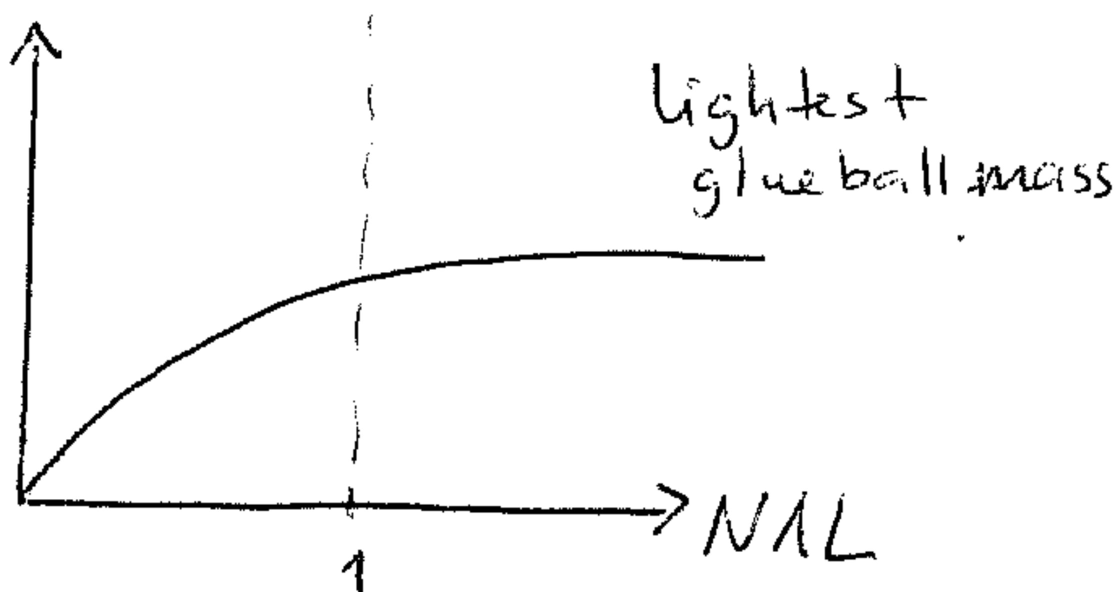
then, $N_{crit} > N_f^*$

true value of critical # "flavors"

thus, for such theories N_f^* is a lower bound thereof

[theory believed to be in this class: QCD(F) - arguments using mixed reps., experiment (lattice)]

C.) if, as # species has not yet reached N_f^*



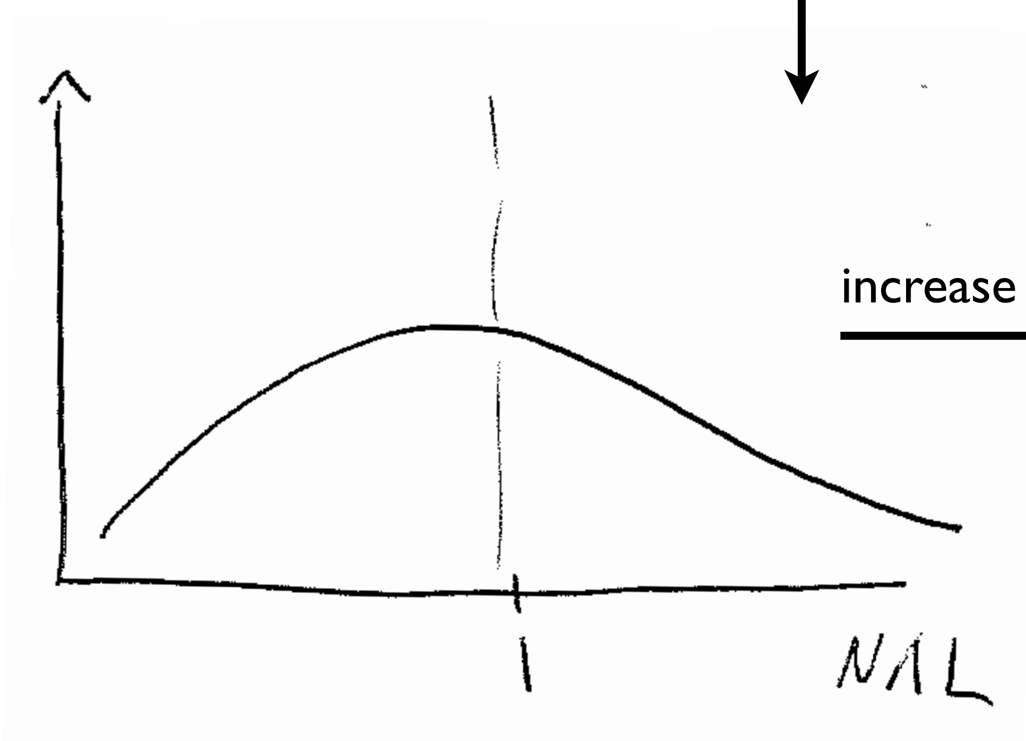
then,

$N_{crit} < N_f^*$

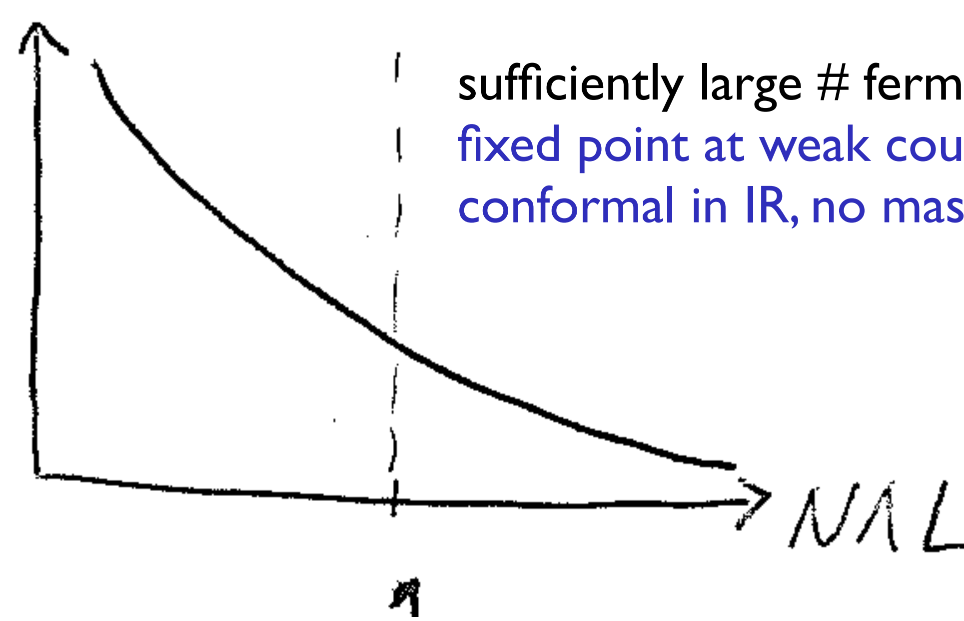
thus, for this class of theories N_f^* is an upper bound on critical # "flavors"

sufficiently small # fermion species
confining theories

increase # fermions



increase # fermions



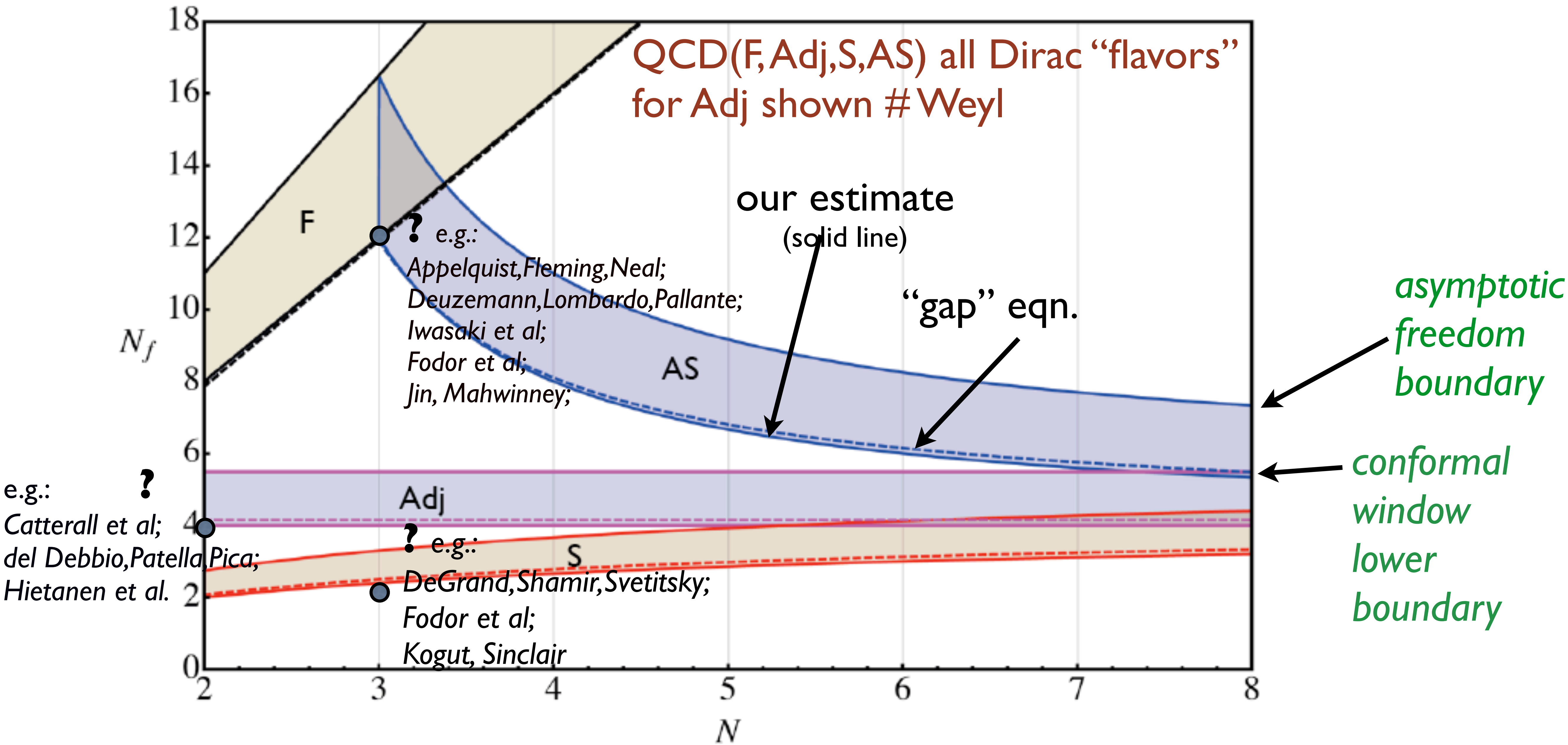
sufficiently large # fermion species
fixed point at weak coupling
conformal in IR, no mass gap

[only one theory we know is believed to be in this class: SU(2) 4-index symmetric tensor Weyl, theory arguments]

“experimental” results (lattice) | yet uncertain: *finite V*, *chiral limit*, *$$$*

vs

theory estimates | *error bars unknown*



gap equation and lattice - only vectorlike theories

in chiral gauge theories - our estimates are the only known ones, save for Sannino’s recent 0911.0931 via the proposed exact beta function [we agree and disagree (mostly)] this - largely (given the absence of credible error bars) - agreement is, to us, somewhat amusing...

compare the tools used:

gap equation

conformality tied to absence of chiral symmetry breaking

compares fixed-point coupling to critical gauge coupling for chiral symmetry breaking - *ladder diagram "approximation" of truncated Schwinger-Dyson eqns. for fermion propagator in Landau gauge* - must use at least 2-loop beta function to get fixed-point g

our estimate

conformality tied to absence of mass gap/string tension - see also Armoni, 2009 (worldine approach; very similar numbers)

semiclassical analysis on a non-thermal circle

dilution vs. non-dilution of topological excitations with L

use only 1-loop beta function

lattice

in principle, (modulo V, m, $...) a first-principle determination

Conclusions 1:

Compactifying 4d gauge theories on a small circle is a "deformation" where nonperturbative dynamics is under control - dynamics as "friendly" as in SUSY, e.g. Seiberg-Witten.

Confinement is due to various "oddball" topological excitations, in most theories non-self-dual.

Polyakov's "Debye screening" mechanism works on $R^3 \times S^1$ also with massless fermions, contrary to what many thought - KK monopoles and index theorem-crucial ingredients of analysis.

Precise nature - monopoles, bions, triplets, or quintets **- depends on the light fermion content of the theory.**

U,P; 0812.2085, 0906.5156

Conclusions II:

didn't have time for these:

Found chiral symmetry breaking (Abelian) due to expectation values of topological "disorder" operators: occurs in mixed-rep. theories with anomaly-free chiral U(1), broken at any radius

U,P; 0910.1245

Circle compactification gives another calculable deformation of SUSY theories - not yet fully explored -

in I=3/2 SU(2) Intriligator-Seiberg-Shenker model we argued that theory conformal, rather than SUSY-breaking.

U,P; 0905.0634

Conclusions III:

Gave "estimates" of conformal window boundary in vectorlike and chiral gauge theories (OK with "experiment" when available). Conformality tied to relevance vs irrelevance of topological excitations.

U,P; 0906.5156

(further similarity to KT transition? Kaplan, Son, Stephanov, 2009-in 2d, vortices proliferate in high-T phase and irrelevant in conformal phase)

It is not so crazy to expect "relevance vs. irrelevance" also in R^4:

Lattice studies in pure YM (early ref.: Kronfeld et al, 1987) have found that confinement appears to be due to topological excitations- center vortices, monopoles - and the deconfinement transition is associated with them becoming irrelevant (large literature...) .

To expect that massless fermions would affect the nature of topological excitations is also quite natural. What is harder (for me) is to make this precise at large L.

back-up slide

Found chiral symmetry breaking (Abelian) due to expectation values of topological “disorder” operators.

- didn’t have time for this (occurs in mixed-rep. theories)

Example:

Unsal,EP 0910.1245

$(N_{\rm adj}{=}1,\ N_{\rm F}{=}1)\ SU(2)$

	$U(1)_B$	$U(1)_A$
λ	0	1
ψ_L	1	-2
ψ_R	-1	-2
$e^{i\sigma}$	0	-2

$(N_{\rm adj}, N_{\rm F}) = (1,1)$: $\quad \mathbb{R}^3 \quad \langle e^{i\sigma}\rangle = 1 \quad \longrightarrow \quad \langle\lambda\lambda\rangle = \Lambda^3 \quad \mathbb{R}^4 \quad L$

small-L: disorder operator vev
Goldstone is dual photon

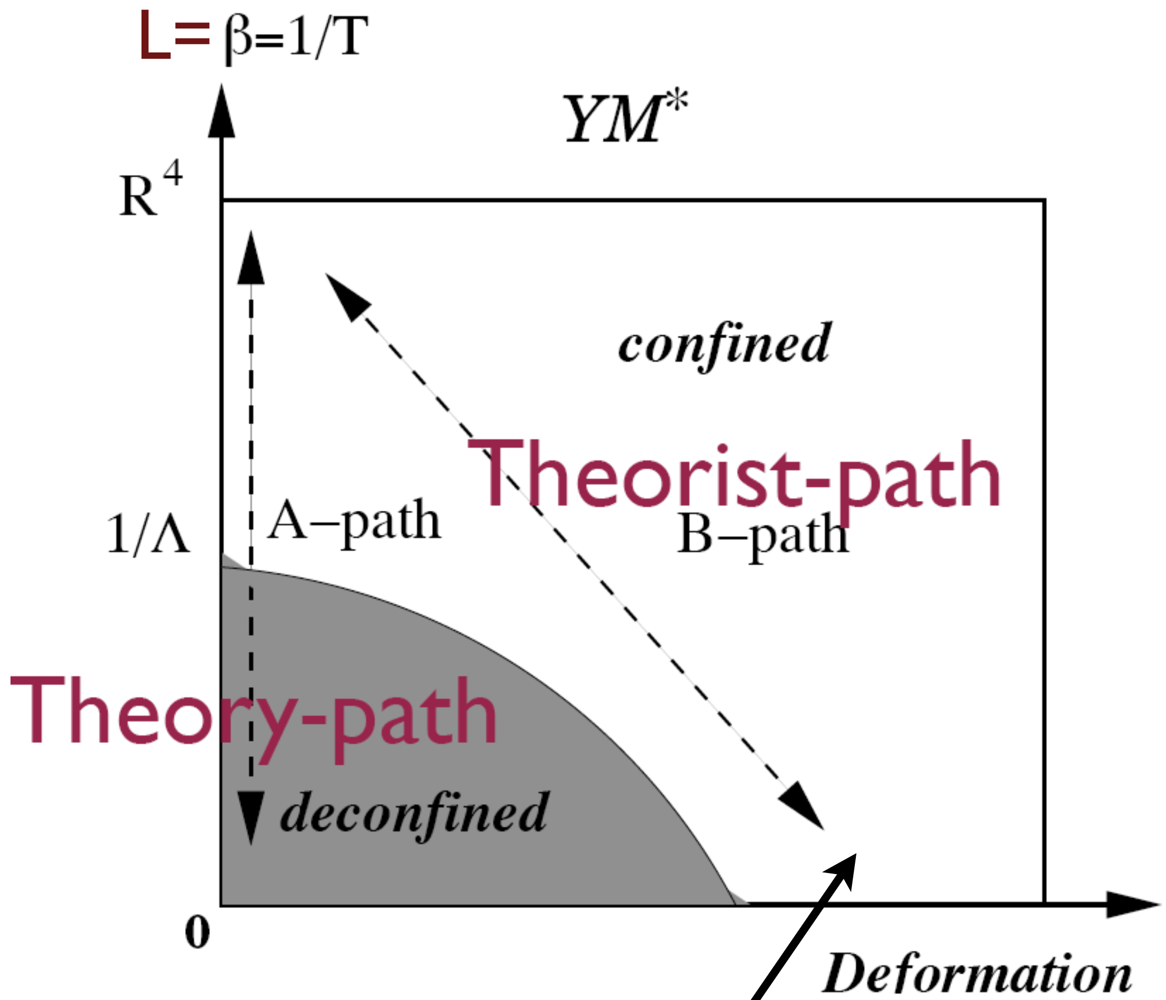
$$\mathcal{L}^{\rm dual} \supset e^{-S_0}\langle e^{i\sigma}\rangle\lambda^2 + e^{-2S_0}\langle e^{-2i\sigma}\rangle\psi_L\psi_R$$

$$\langle\lambda\lambda\rangle = \Lambda^3 e^{i\pi/f_\pi}, \qquad \langle\psi_L\psi_R\rangle = \Lambda^3 e^{-2i\pi/f_\pi}$$

large-L: fermion condensate
Goldstone is “pion-like”

small-L and large-L regimes can smoothly merge via NJL-like breaking
due to monopole operators becoming strong at L ~ Lambda

II.:"Deformation Theory"-needed, e.g., in QCD with fundamentals, not needed in QCD with adjoints, SUSY, etc...



$$S^{\mathrm{YM}^*} = S^{\mathrm{YM}} + \int_{R^3 \times S^1} P[U(\mathbf{x})]$$

$$P[U] = A \frac{2}{\pi^2 \beta^4} \sum_{n=1}^{\lfloor N/2 \rfloor} \frac{1}{n^4} |\mathrm{tr}\,(U^n)|^2$$

double-trace deformations

theory is under control here:
can calculate mass gap for gauge fluctuations, string tensions (as in Seiberg-Witten theory)

decompactification smooth in the sense of center symmetry

- lattice studies back up smoothness in some models (Ogilvie, Myers, Meisinger, 2008)
- interesting at large-N: P[U] ensures center symmetry but decouples from observables... in the volume-independence context (Unsal, Yaffe, 2008)

---

in what follows, we assume center-symmetric vacuum - due to either I. or II.